THE KINETICS OF SOFTENING OF MATERIALS IN NONSTATIONARY REGIME OF HEATING

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Equations for determining the working life of a plate subjected to a longitudinal force in the presence of symmetric heating are derived taking into account the change of the modulus of elasticity and the ultimate strength of the material with temperature.

The reason of a gradual softening of products on heating is the nonstationary process of change of the mechanical properties of the material, which occurs as the heat front moves deeper into the product. An additional lowering of the strength produces thermal stresses which appear in the presence of large temperature gradients. In view of this it is of practical interest to estimate the decrease of the working life of products, subjected to heating, using traditional criteria applied in heat-strength computations, for example, the decrease of the safety factor or the carrying capacity.

We shall derive the desired equations in the simplest case of loading, i.e., for a uniaxial stretching (compression) of a very thin plate-sample ($H \ll 2R$) and of a plate, whose width is comparable to the thickness; the heat flux is one-dimensional and directed perpendicular to the applied force. Initially we assume that the material obeys Hooke's law and its ultimate strength is constant, being independent of the time of loading.

For a symmetric thermal action on the thin plate the stresses in the transverse direction can be neglected; then the stresses at any point in the plate are given by [1]

$$\sigma_x = E\,[(\overline{\sigma_x} - \overline{E\alpha\Delta T})/E - \alpha\Delta T]. \tag{1}$$

In this expression the bar on the top denotes integral mean values of the corresponding parameters over the thickness of the plate.

The softening of samples on heating is characterized by the relative strength $n_t = (\sigma_x)_p/\sigma_b(T_0)$, where $\sigma_b(T_0)$ is the ultimate strength at $T = T_0$. From Eq. (1) for α = const we have

$$n_t = \overline{E}/E - \alpha\,\overline{(E\Delta T} - \overline{E}\,\overline{\Delta T})/\sigma_b = n_{tE} + n_{t\alpha}. \tag{2}$$

The first term in this formula shows the factor by which the relative strength is lowered due to the decrease in the modulus of elasticity of the material at increased temperature; the second characterizes the decrease in the working capacity only due to the appearance of thermal stresses if $\alpha = 0$, $n_{t\alpha} = 0$.

For making the dependence $n_t(t)$ specific it is necessary to approximate the experimental graph $E(T)$ of the given material by an analytic function. In this respect the most convenient are the functions:

a) the linear function

$$E(T) = E_0\,[1 - \zeta(T - T_0)]; \tag{3}$$

b) the step function

$$E(T) = E_0 \text{ for } T \leqslant T^* \text{ and } E = 0 \text{ for } T > T^*; \tag{4}$$

c) the hyperbolic function

$$E(T) = E_0/[1 + \xi(T - T_0)]; \tag{5}$$

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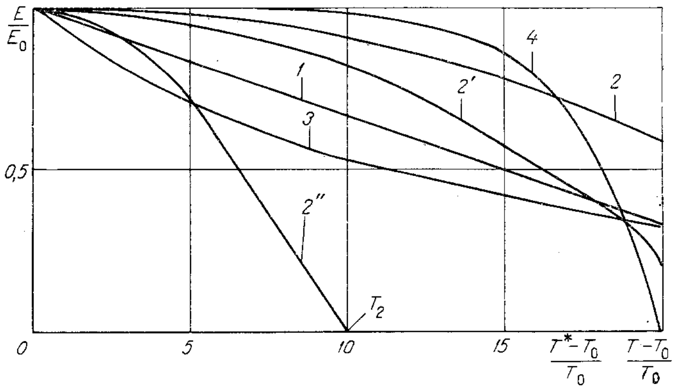

Fig. 1. Different methods of approximation of the experimental dependence E (T): 1) by Eq. (3); 2) by Eq. (6) (2 for $\varkappa$ = 0.001, 2' for $\varkappa$ = 0.002, 2" for $\varkappa$ = 0.01); 3) by Eq. (5), T = 20°C, ξ = 0.1; 4) experimental data.

d) the parabolic function

$$E(T)=E_0[1-\varkappa(T-T_0)^2]. \tag{6}$$

The graphs of these functions, referred to E_0, are shown in Fig. 1 together with the characteristic dependence E(T). We shall make use of functions (3)-(6) in the determination of the terms n_{tE} and $n_{t\alpha}$ of formula (2) in the case of a linear law of variation of the surface temperature of the plate:

$$T(R,\,t)=T_0+bt=T_0(1+\mathrm{PdFo}). \tag{7}$$

For the linear approximation of the dependence E(T) we have

$$n_{tE}=E_0[1-\zeta(T-T_0)]/E. \tag{8}$$

It follows from here that in order to compute n_{tE} it is sufficient to know the variation of the average temperature of the plate with time. The average temperature of the plate for the adopted law of heating is given by the relation [2]

$$\frac{\overline{T}-T}{T_0}=\mathrm{Pd}\left[\mathrm{Fo}-\frac{1}{3}+\sum_{n=1}^{\infty}\frac{B_n}{\mu_n^2}\exp(-\mu_n^2\mathrm{Fo})\right]. \tag{9}$$

For small values of Fo number from the last equation we obtain

$$n_{tE}=1-\zeta\frac{bt_\mathrm{p}\sqrt{(\mathrm{Fo})_\mathrm{p}}}{3\sqrt{\pi}}=1-\zeta\,T_0\,\mathrm{Pd(Fo)_p}\quad\frac{\sqrt{(\mathrm{Fo})_\mathrm{p}}}{3\sqrt{\pi}} \tag{10}$$

and correspondingly for large Fo numbers (quasistationary regime of heating)

$$n_{tE}=\frac{E_0\sigma_b(T)}{E\sigma_b(T_0)}\left[1-\zeta b\left(t_\mathrm{p}-\frac{R^2}{3a}\right)\right]=\frac{E_0}{E}\left\{1-\zeta T_0\,\mathrm{Pd}\left[(\mathrm{Fo})_\mathrm{p}-\frac{1}{3}\right]\right\}n_\sigma. \tag{11}$$

The working life of the plate for a given relative strength n_{tE} is easily determined from the last equation.

A more accurate description of the temperature dependence E(T) than the linear function is given by the parabolic function (6); using this function we get $n_{tE}=\overline{E}/E=\varkappa(T-T_0)^2$. Calculating the value of $(T-T_0)^2$ we obtain

$$n_{tE}=\frac{E_0}{E}\left[1-\varkappa T_0^2\,\mathrm{Pd}^2\,(\mathrm{Fo})^2-\left(\frac{2}{3}\mathrm{Fo}+\frac{2}{15}\right)\right]. \tag{12}$$

The computation of n_{tE} with the use of functions (3) and (6) is possible only under the conditions $T \leq T_1 = 1/\xi + T_0$ for the linear and $T \leq T = \sqrt{1/\varkappa} + T_0$ for the parabolic approximation of the dependence E(T). If these conditions are not satisfied, substitution of the variable T in formulas (3), (6) gives negative values of the modulus of elasticity.

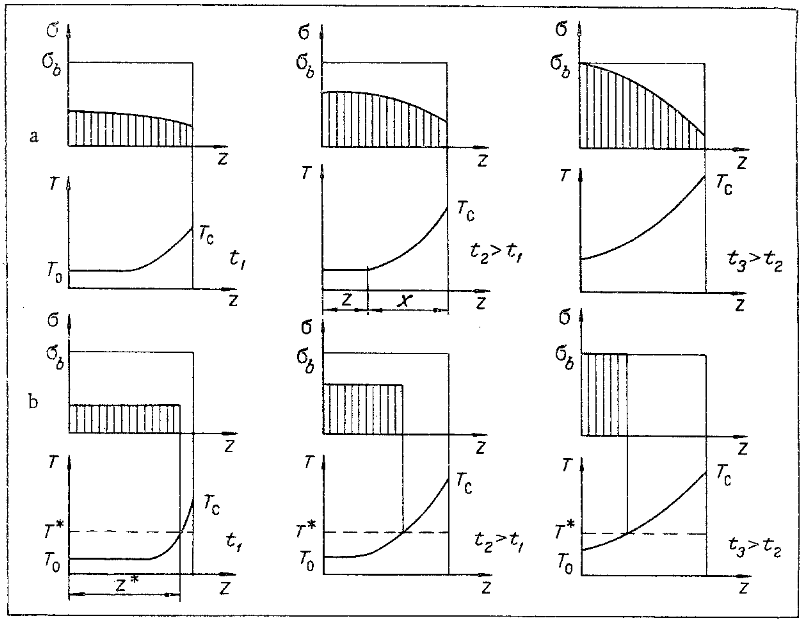

Fig. 2. Redistribution of stresses over the plate thickness, corresponding to the linear approximation (a) and step-function approximation (b) of the dependence E(T).

For many compound materials the step-function approximation (4) is valid for high-temperature heating. The limiting temperature T* approximately corresponds to the temperature of the most intensive softening of the thermoplastic components binding the composition materials. The step-function approximation of the dependence E(T) is equivalent to the assertion that the displacement of the isothermal surfaces with temperature T* leads to the exclusion of the layer, heated to higher temperatures and lying beyond this surface (see Fig. 2), from operation. The relative deformations of such layers of the material are infinitely large: $\varepsilon_x = (\sigma/E)_{E\to 0} \to \infty$; therefore the temperature dependence $\sigma_b(T)$ of the material of these layers has no effect on the final result of computation of n_t. This step-by-step softening of the layers of the plate can be represented as a decrease of its thickness with the rate of displacement of the temperature front T*. Hence the softening of the plate begins at the instant when the stresses in its heated part, where T < T*, reach the ultimate strength

$$\sigma_b \cdot 2 \cdot z^* = (\bar{\sigma}_x)_p \cdot 2 \cdot R = P_p/H. \tag{13}$$

Whence we obtain

$$n_{tE} = z^*/R. \tag{14}$$

Closed exact solutions in these methods of approximation can be obtained only for a very small number of heating regimes of the plate. Thus, for example, for a parabolic temperature distribution putting T = T* in the formula

$$T = T_0 + bt - \frac{b}{2a}(R^2 - z^2), \tag{15}$$

and expanding it with respect to z*, we find that

$$n_{tE} = \sqrt{2\,\frac{T^* - T_0}{bR}\,a - 2\,\frac{at}{R^2} + 1} = \sqrt{2K_1 - 2\mathrm{Fo} + 1}. \tag{16}$$

The approximation of E(T) with the use of function (5) leads to the following result:

$$n_{tE} = \frac{1}{2RE}\int_{-R}^{+R} \frac{E_0}{1 + \xi(T - T_0)}\,dz = \frac{E_0}{\sqrt{K_3(1 + \xi bt - K_3)}}\,\mathrm{arctg}\,\frac{K_3}{\sqrt{K_3(1 + \xi bt - K_3)}}. \tag{17}$$

For heating regimes of the plate other than the linear variation of the surface temperature with time approximate methods based on the elimination of the spatial coordinate are very effective. The use of these

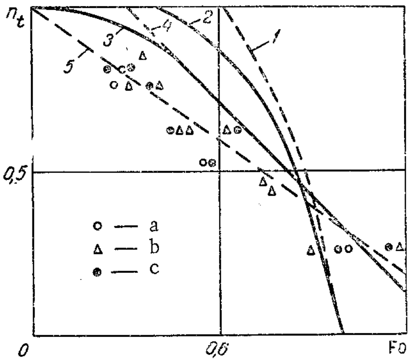

Fig. 3. Kinetics of softening of AG-4S glass-fiber-reinforced plastic specimens under short-term heating with different compression loads: 1) according to (19), without taking into account the dependence of σ_b on T; 2) according to (19), taking into account the temperature dependence of σ_b; 3) according to (10), $T^* = 170°C$; 4) according to (11); 5) experimental data, determined from specimens a) 8, b) 5, and c) 3 mm thick.

methods is of practical interest, since the working life of the plate and the average stress $(\overline{\sigma_x})_p$ can be interrelated through a simple relationship. Thus, for a variation of the surface temperature of the plate by a jump from T_0 to T_c the displacement of a fixed temperature front $\vartheta^* = (T^* - T_0)/(T_c - T_0)$ in the plate is described by the relation

$$R - z = R - K_2 \sqrt{12\,(\mathrm{Fo})_p}\,. \tag{18}$$

Hence

$$n_{te} = 1 - K_2 \sqrt{12\,(\mathrm{Fo})_p}\,. \tag{19}$$

We now turn to the determination of the second term in formula (2) for the parabolic law of temperature distribution in the plate and linear approximation of the dependence E(T). In this case we have $\overline{E\Delta T} = E_0[\overline{(T - T_0)} - \xi\overline{(T - T_0)^2}]$.

Substituting the value of the temperature and averaging, after some manipulations we obtain

$$n_{t\alpha} = -\frac{\alpha E_0 T_0 \mathrm{Pd}}{6\sigma_b}\left\{1 - \zeta\left[(\mathrm{Fo})_p + \frac{1}{5}\right]\mathrm{Pd}\,T_0\right\} \tag{20}$$

in the computation at the middle layer (z = 0) and

$$n_{t\alpha} = +\frac{\alpha E_0 T_0 \mathrm{Pd}}{6\sigma_b}\left\{1 - \zeta\left[(\mathrm{Fo})_p - \frac{2}{5}\right]\mathrm{Pd}\,T_0\right\} \tag{21}$$

in the computation at the surface layer (z = R). Of the two last values of $n_{t\alpha}$ the value giving the smallest coefficient $n_{t\alpha}$ should be taken, i.e., depending on whether the unsafe point lies at the middle or the surface layer of the plate. For a step-function approximation of E(T) we have

$$\overline{E\Delta T} = \frac{E_0 z^*}{R}\left(bt - \frac{bR^2}{2a} + \frac{b}{6a} z^{*2}\right), \tag{22}$$

$$n_t = n_{tE}\left(1 - \frac{\alpha E_0 T_0 \mathrm{Pd}}{6\sigma_b} n_{tE}\right) \tag{23}$$

for z = 0 and

$$n_t = n_{te}\left[1 - \frac{\alpha E_0 T_0 \mathrm{Pd}}{2\sigma_b}\left(\frac{1}{3} n_{te} - 1\right)\right] \tag{24}$$

for z = R.

As an example we use the obtained relations to describe the experimental data of [2] on the kinetics of softening of glassfiber-reinforced plastics under the action of heat fluxes of large intensity. In this work the results of tests of samples, whose surface temperature was varied in accordance with the linear law with the use of a programmed device, are presented. The experimental values of n_t for samples of different thickness (Pd = 1) are presented in Fig. 3 and the curves of $n_t(t)$, constructed from formulas (10) and (11), are plotted. A comparison of the experimental and computational results shows that the linear approximation of the dependence E(T) gives the best agreement: the maximum deviation from the mean value of n_t at Fo = 0.35 does not exceed 20%. Two types of disintegration of the samples were observed in the tests: laminar disintegration which started at $n_t < 0.5$, and brittle disintegration which started at $n_t > 0.5$. In this connection let us turn back to formula (24), from which it follows that at small values of n_t the second term, which determines the level of the thermal stresses, is comparable with the term n_{tE} in order of magnitude. In the experiment the tested samples were subjected to compression; therefore for small n_t the thermal stresses combine additively with the stresses σ_x from the external force and produce a peak of the stresses in the surface zone of heating. As the photographs showed, this led to a local loss of stability of the surface layers. According to formula (21) the parameter $n_{t\alpha}$ increases in proportion to the rate of heating and is independent of σ_x. Hence the increase of n_t is caused by the increase of the first term n_{tE}. This means that with the change in n_t the maximum of the stress diagram shifts toward the middle surface and the sample disintegrates in all the layers following the disintegration of the middle carrying layer. Thus the softening of the samples occurs in two stages which remain undetected in the analysis by the methods of the theory of similarity. The first of these is responsible for the appearance and development of thermal stresses, the second for their equalization. The disintegration of the samples starts (depending on the mechanical and thermophysical properties of the material) either at the instant of the action of maximum thermal stresses at the end of the first stage, or as a result of vanishing of the carrying capacity of the plate at the end of the second stage.

The initial assumption that σ_b and E are constant quantities is not always valid for real materials, especially at increased temperatures. Computation of long-term strength $\sigma_b(T, t)$ and durability are carried out based on the principle of superposition of damage-liabilities of the material using Baily's criterion [3]. Another possible method of computing the change of temperature and stresses in the calculation of long-term strength is based on the representations of the activation theory of strength [4, 5]. According to this theory the disintegration of bodies is explained by the rupture of chemical bonds, occurring with the rate

$$k = k_0 \exp(-U/RT). \tag{25}$$

In the case of a linear dependence of the activation energy on the stress, $U = U_0 - \gamma\sigma$, t_p is determined by Eq. (27). Assuming that the ultimate strength of the material is proportional to the number of bonds N and their disintegration obeys the first order reaction equation $dN/dt = -kN$, by integrating we obtain the following relation between the disintegrating stress and the duration of its action

$$\sigma_b = \sigma_\infty + (\sigma_{\text{ins}} - \sigma_\infty)\exp \int_0^{t_p} k dt. \tag{26}$$

In practice it is sometimes important to obtain an approximate estimate of the smallest value of the working life of the material in the unsafe layer of the heated plate. Then, if the stress in the unsafe layer is amonotonic function of time, using the mean value theorem we obtain

$$\int_0^{t_p} k dt = t_p k_{\text{av}} = \frac{k_0}{2}\left[\exp\left(-\frac{U_0 - \gamma\sigma_0}{RT_0}\right) + \exp\left(-\frac{U_0 - \gamma\sigma_b}{RT}\right)\right]. \tag{27}$$

Taking into account the variation of the ultimate strength of the material with temperature and time

$$n_t' = n_t \sigma_b(T, t)/\sigma_b(T). \tag{28}$$

The effect of the temperature–time dependence of elastic and viscous-elastic properties of the material on the stressed state of the sample in nonstationary regime of heating is discussed in [7].

The relations obtained here can be generalized to the case of heating of a plate of large width, if the Poisson coefficient of the material is constant. Then, as shown in [1], in the plane stressed state

$$\sigma_x = \frac{E}{1 - \nu^2}\left[\varepsilon_x + \nu\varepsilon_y - \alpha T(1 + \nu)\right] \tag{29}$$

and, hence,

$$n_t = n_{te} + n_{t\alpha}/(1 - \nu). \tag{30}$$

NOTATION

H	is the width of the plate;
2R	is the thickness of the plate;
$(\overline{\sigma}_x)_p$	is the mean stress at which disintegration of the plate occurs;
$Pd = bR/aT_0$	is the Predvoditelev number;
$b = dT/dt$	is the rate of heating;
$\mu_n = (2n - 1) \cdot \pi/2$	are the characteristic numbers;
A, B	are the constant coefficients;
$(Fo)_p$	is the Fourier number corresponding to the instant of disintegration of the sample;
z^*	is the coordinate of the layer with temperature T^*;

$K_1 = (T/T_0 - 1)/Pd$;

$K_2 = 1 - \sqrt{(T^* - T_0)/(T_c - T_0)}$;

$K_3 = \xi b R^2/2$;

ν is the Poisson coefficient;

k_0, U_0, γ are material constants;

σ_{ins}, σ_∞ are the instantaneous and equilibrium values of disintegration stresses.

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